6.7 Areas of Triangles and Quadrilaterals

What you should learn

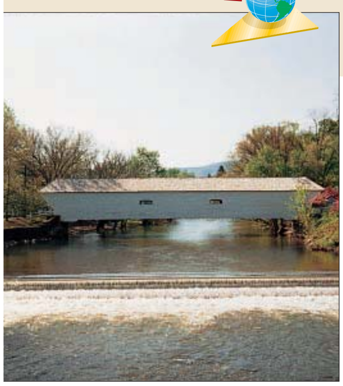
GOAL 1 Find the areas of squares, rectangles, parallelograms, and triangles.

GOAL 2 Find the areas of trapezoids, kites, and rhombuses, as applied in **Example 6**.

Why you should learn it

▼ To find areas of **real-life** surfaces, such as the roof of the covered bridge in **Exs. 48 and 49**.

REAL LIFE



GOAL 1 USING AREA FORMULAS

You can use the postulates below to prove several area theorems.

AREA POSTULATES

POSTULATE 22 ***Area of a Square Postulate***

The area of a square is the square of the length of its side, or $A = s^2$.

POSTULATE 23 ***Area Congruence Postulate***

If two polygons are congruent, then they have the same area.

POSTULATE 24 ***Area Addition Postulate***

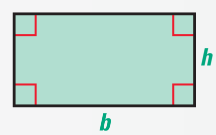
The area of a region is the sum of the areas of its nonoverlapping parts.

AREA THEOREMS

THEOREM 6.20 ***Area of a Rectangle***

The area of a rectangle is the product of its base and height.

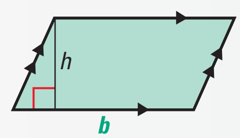
$$A = bh$$



THEOREM 6.21 ***Area of a Parallelogram***

The area of a parallelogram is the product of a base and its corresponding height.

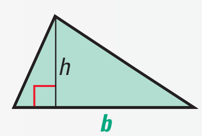
$$A = bh$$



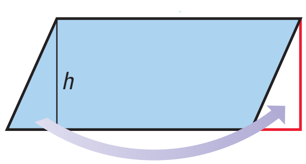
THEOREM 6.22 ***Area of a Triangle***

The area of a triangle is one half the product of a base and its corresponding height.

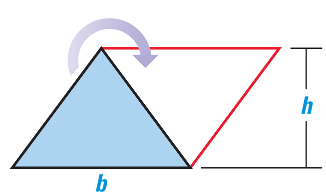
$$A = \frac{1}{2}bh$$



You can justify the area formulas for triangles and parallelograms as follows.



The area of a parallelogram is the area of a rectangle with the same base and height.

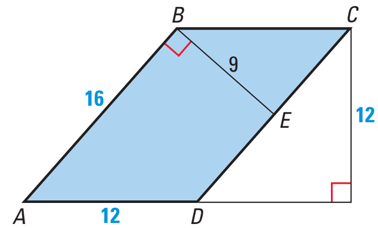


The area of a triangle is half the area of a parallelogram with the same base and height.

> **STUDENT HELP**
>
> **Study Tip**
> To find the area of a parallelogram or triangle, you can use any side as the base. But be sure you measure the height of an altitude that is perpendicular to the base you have chosen.

EXAMPLE 1 *Using the Area Theorems*

Find the area of $\square ABCD$.



SOLUTION

Method 1 Use $\overline{AB}$ as the base. So, $b = 16$ and $h = 9$.

Area $= bh = 16(9) = 144$ square units.

Method 2 Use $\overline{AD}$ as the base. So, $b = 12$ and $h = 12$.

Area $= bh = 12(12) = 144$ square units.

Notice that you get the same area with either base.

Using Algebra

EXAMPLE 2 *Finding the Height of a Triangle*

Rewrite the formula for the area of a triangle in terms of h. Then use your formula to find the height of a triangle that has an area of 12 and a base length of 6.

SOLUTION

Rewrite the area formula so h is alone on one side of the equation.

$A = \frac{1}{2}bh$ **Formula for the area of a triangle**

$2A = bh$ **Multiply both sides by 2.**

$\frac{2A}{b} = h$ **Divide both sides by *b*.**

Substitute **12** for A and **6** for b to find the height of the triangle.

$$h = \frac{2A}{b} = \frac{2(12)}{6} = 4$$

▶ The height of the triangle is 4.

EXAMPLE 3 *Finding the Height of a Triangle*

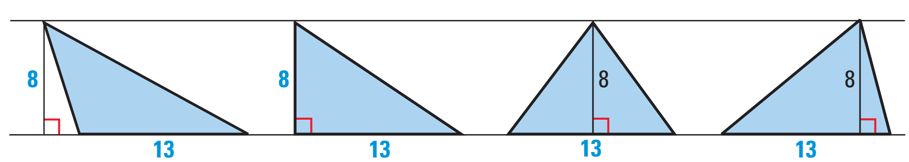
A triangle has an area of 52 square feet and a base of 13 feet. Are all triangles with these dimensions congruent?

SOLUTION

Using the formula from Example 2, the height is $h = \frac{2(52)}{13} = 8$ feet.

There are many triangles with these dimensions. Some are shown below.

> **STUDENT HELP**
>
> **Study Tip**
> Notice that the altitude of a triangle can be outside the triangle.



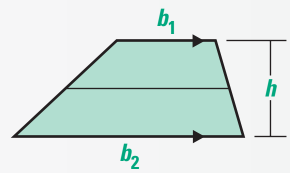
GOAL 2 AREAS OF TRAPEZOIDS, KITES, AND RHOMBUSES

THEOREMS

THEOREM 6.23 ***Area of a Trapezoid***

The area of a trapezoid is one half the product of the height and the sum of the bases.

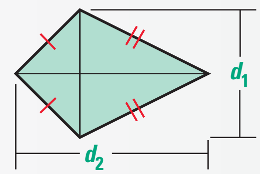
$$A = \frac{1}{2}h(b_1 + b_2)$$



THEOREM 6.24 ***Area of a Kite***

The area of a kite is one half the product of the lengths of its diagonals.

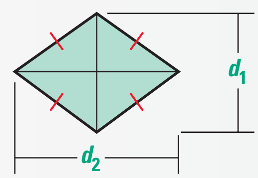
$$A = \frac{1}{2}d_1d_2$$



THEOREM 6.25 ***Area of a Rhombus***

The area of a rhombus is equal to one half the product of the lengths of the diagonals.

$$A = \frac{1}{2}d_1d_2$$

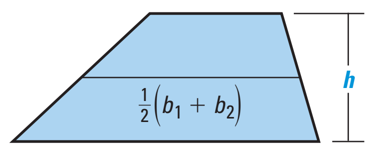


STUDENT HELP

Look Back
Remember that the length of the midsegment of a trapezoid is the average of the lengths of the bases. (p. 357)

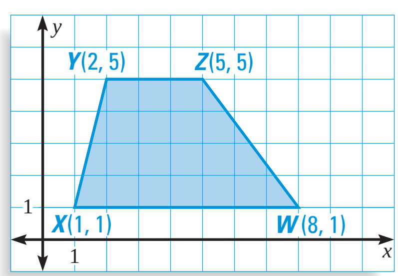
You will justify Theorem 6.23 in Exercises 58 and 59. You may find it easier to remember the theorem this way.

Area = **Length of Midsegment** • **Height**



EXAMPLE 4 *Finding the Area of a Trapezoid*

Find the area of trapezoid *WXYZ*.



SOLUTION

The height of *WXYZ* is $h = 5 - 1 = 4$.

Find the lengths of the bases.

$$b_1 = YZ = 5 - 2 = 3$$

$$b_2 = XW = 8 - 1 = 7$$

Substitute 4 for h, 3 for b_1, and 7 for b_2 to find the area of the trapezoid.

$A = \frac{1}{2}h(b_1 + b_2)$ **Formula for area of a trapezoid**

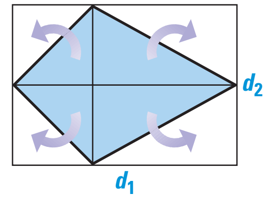
$= \frac{1}{2}(4)(3 + 7)$ **Substitute.**

$= 20$ **Simplify.**

▶ The area of trapezoid *WXYZ* is 20 square units.

The diagram at the right justifies the formulas for the areas of kites and rhombuses.

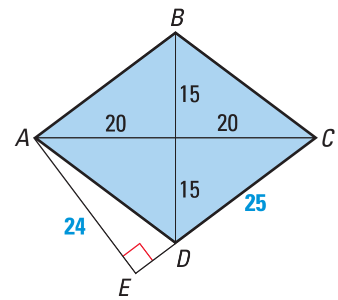
The diagram shows that the area of a kite is half the area of the rectangle whose length and width are the lengths of the diagonals of the kite. The same is true for a rhombus.



$A = \frac{1}{2}d_1d_2$

EXAMPLE 5 *Finding the Area of a Rhombus*

Use the information given in the diagram to find the area of rhombus *ABCD*.



SOLUTION

Method 1 Use the formula for the area of a rhombus. $d_1 = BD = 30$ and $d_2 = AC = 40$.

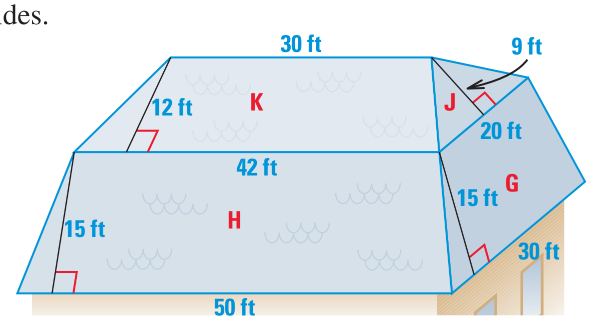
$$A = \frac{1}{2}d_1d_2$$
$$= \frac{1}{2}(30)(40)$$
$$= 600 \text{ square units}$$

Method 2 Use the formula for the area of a parallelogram. $b = 25$ and $h = 24$.

$$A = bh = 25(24) = 600 \text{ square units}$$

EXAMPLE 6 *Finding Areas*

REAL LIFE **ROOF** Find the area of the roof. *G*, *H*, and *K* are trapezoids and *J* is a triangle. The hidden back and left sides of the roof are the same as the front and right sides.



SOLUTION

Area of $J = \frac{1}{2}(20)(9) = 90 \text{ ft}^2$ Area of $H = \frac{1}{2}(15)(42 + 50) = 690 \text{ ft}^2$

Area of $G = \frac{1}{2}(15)(20 + 30) = 375 \text{ ft}^2$ Area of $K = \frac{1}{2}(12)(30 + 42) = 432 \text{ ft}^2$

The roof has two congruent faces of each type.

Total Area $= 2(90 + 375 + 690 + 432) = 3174$

▶ The total area of the roof is 3174 square feet.

STUDENT HELP

Study Tip
To check that the answer is reasonable, approximate each trapezoid by a rectangle. The area of *H* should be less than 50 • 15, but more than 40 • 15.

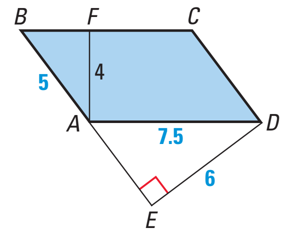
Guided Practice

Vocabulary Check ✓

1. What is the *midsegment* of a trapezoid?

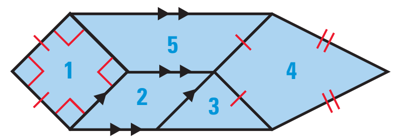
Concept Check ✓

2. If you use AB as the base to find the area of $\square ABCD$ shown at the right, what should you use as the height?



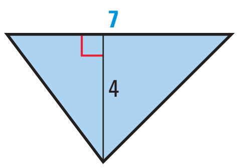
Skill Check ✓

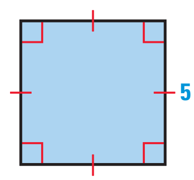
Match the region with a formula for its area. Use each formula exactly once.

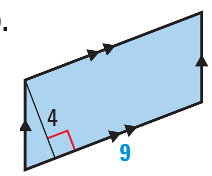


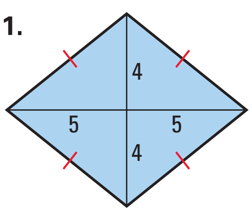
3. Region 1 — **A** $A = s^2$
4. Region 2 — **B** $A = \frac{1}{2}d_1d_2$
5. Region 3 — **C** $A = \frac{1}{2}bh$
6. Region 4 — **D** $A = \frac{1}{2}h(b_1 + b_2)$
7. Region 5 — **E** $A = bh$

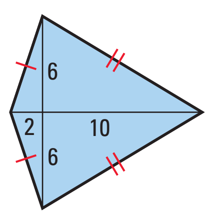
Find the area of the polygon.

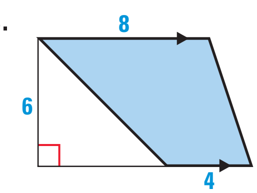
8. 

9. 

10. 

11. 

12. 

13. 

Practice and Applications

Student Help

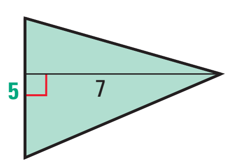
Extra Practice to help you master skills is on p. 814.

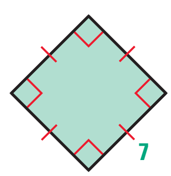
Student Help

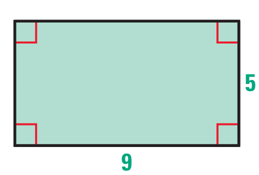
HOMEWORK HELP
Example 1: Exs. 14–19, 41–47
Example 2: Exs. 26–31

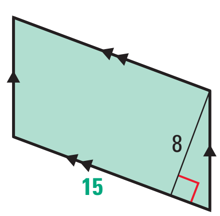
continued on p. 377

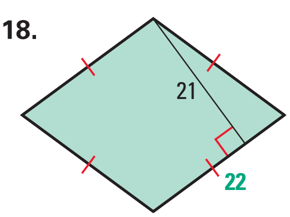
FINDING AREA **Find the area of the polygon.**

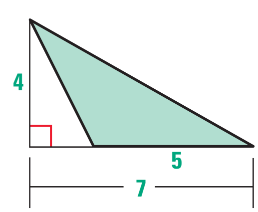
14. 

15. 

16. 

17. 

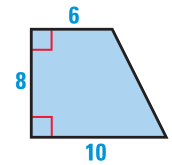
18. 

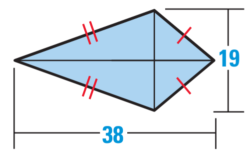
19. 

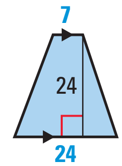
STUDENT HELP

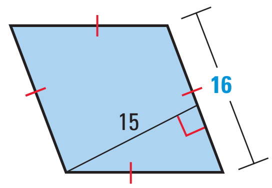
HOMEWORK HELP
continued from p. 376
Example 3: Exs. 26–28, 39, 40
Example 4: Exs. 32–34
Example 5: Exs. 20–25, 44
Example 6: Exs. 35–38, 48–52

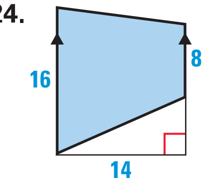
FINDING AREA Find the area of the polygon.

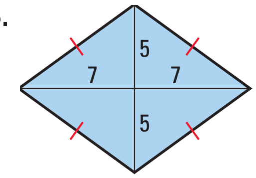
20.


21.


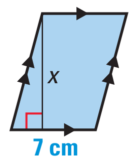
22.


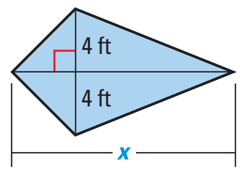
23.


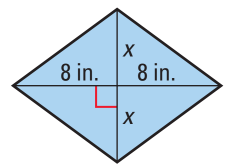
24.


25.


USING ALGEBRA Find the value of x.

26. $A = 63 \text{ cm}^2$


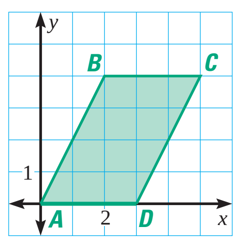
27. $A = 48 \text{ ft}^2$


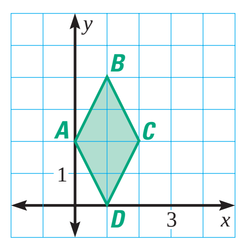
28. $A = 48 \text{ in.}^2$


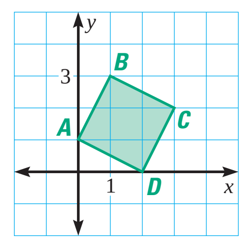
REWRITING FORMULAS Rewrite the formula for the area of the polygon in terms of the given variable. Use the formulas on pages 372 and 374.

29. triangle, b **30.** kite, d_1 **31.** trapezoid, b_1

FINDING AREA Find the area of quadrilateral *ABCD*.

32.


33.


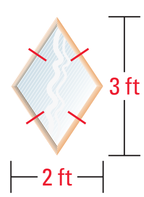
34.


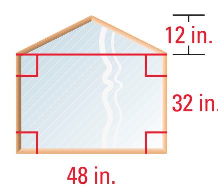
FOCUS ON APPLICATIONS

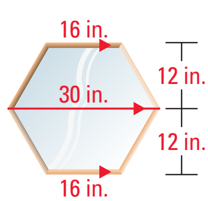
INSULATION Insulation makes a building more energy efficient. The ability of a material to insulate is called its *R*-value. Many windows have an *R*-value of 1. Adobe has an *R*-value of 11.9.

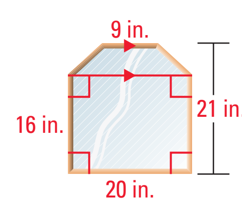
APPLICATION LINK
www.mcdougallittell.com

ENERGY CONSERVATION The total area of a building's windows affects the cost of heating or cooling the building. Find the area of the window.

35.


36.


37.


38.


STUDENT HELP

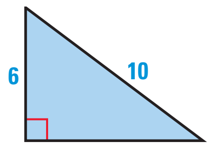
Study Tip
Remember that two polygons are congruent if their corresponding angles and sides are congruent.

39. **LOGICAL REASONING** Are all parallelograms with an area of 24 square feet and a base of 6 feet congruent? Explain.

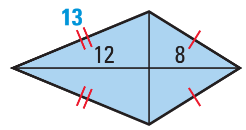
40. **LOGICAL REASONING** Are all rectangles with an area of 24 square feet and a base of 6 feet congruent? Explain.

USING THE PYTHAGOREAN THEOREM **Find the area of the polygon.**

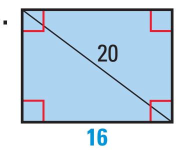
41.



42.



43.



44. **LOGICAL REASONING** What happens to the area of a kite if you double the length of one of the diagonals? if you double the length of both diagonals?

PARADE FLOATS **You are decorating a float for a parade. You estimate that, on average, a carnation will cover 3 square inches, a daisy will cover 2 square inches, and a chrysanthemum will cover 4 square inches. About how many flowers do you need to cover the shape on the float?**

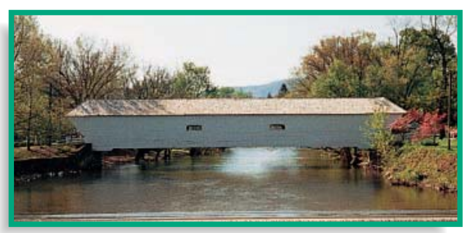
45. Carnations: 2 ft by 5 ft rectangle

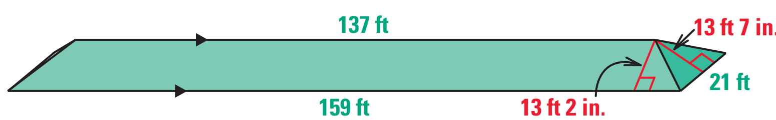
46. Daisies: trapezoid ($b_1 = 5$ ft, $b_2 = 3$ ft, $h = 2$ ft)

47. Chrysanthemums: triangle ($b = 3$ ft, $h = 8$ ft)

BRIDGES **In Exercises 48 and 49, use the following information.**

The town of Elizabethton, Tennessee, restored the roof of this covered bridge with cedar shakes, a kind of rough wooden shingle. The shakes vary in width, but the average width is about 10 inches. So, on average, each shake protects a 10 inch by 10 inch square of roof.

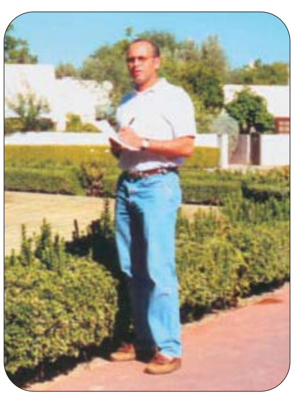




48. In the diagram of the roof, the hidden back and left sides are the same as the front and right sides. What is the total area of the roof?

49. Estimate the number of shakes needed to cover the roof.

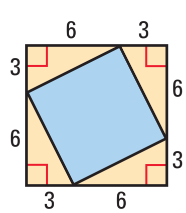
FOCUS ON PEOPLE



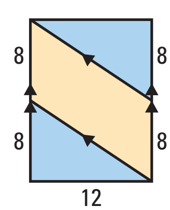
REAL LIFE **MARK CANDELARIA**
When Mark Candelaria restored historic buildings in Scottsdale, Arizona, he calculated the areas of the walls and floors that needed to be replaced.

AREAS **Find the areas of the blue and yellow regions.**

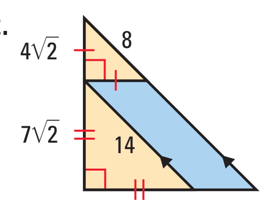
50.



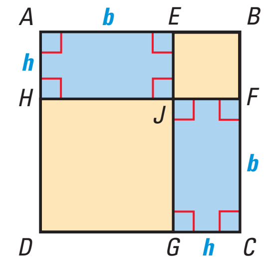
51.



52.



JUSTIFYING THEOREM 6.20 **In Exercises 53–57, you will justify the formula for the area of a rectangle. In the diagram, *AEJH* and *JFCG* are congruent rectangles with base length *b* and height *h*.**



53. What kind of shape is *EBFJ*? *HJGD*? Explain.

54. What kind of shape is *ABCD*? How do you know?

55. Write an expression for the length of a side of *ABCD*. Then write an expression for the area of *ABCD*.

56. Write expressions for the areas of *EBFJ* and *HJGD*.

STUDENT HELP

Look Back
For help with squaring binomial expressions, see p. 798.

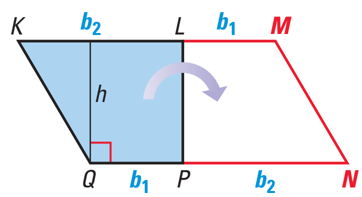
57. Substitute your answers from Exercises 55 and 56 into the following equation. Let A = the area of *AEJH*. Solve the equation to find an expression for A.

Area of *ABCD* = Area of *HJGD* + Area of *EBFJ* + 2(Area of *AEJH*)

JUSTIFYING THEOREM 6.23 **Exercises 58 and 59 illustrate two ways to prove Theorem 6.23. Use the diagram to write a plan for a proof.**

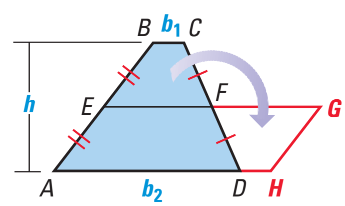
58. GIVEN ▶ *LPQK* is a trapezoid as shown. *LPQK* ≅ *PLMN*.

PROVE ▶ The area of *LPQK* is $\frac{1}{2}h(b_1 + b_2)$.



59. GIVEN ▶ *ABCD* is a trapezoid as shown. *EBCF* ≅ *GHDF*.

PROVE ▶ The area of *ABCD* is $\frac{1}{2}h(b_1 + b_2)$.



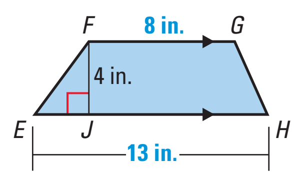
Test Preparation

60. MULTIPLE CHOICE What is the area of trapezoid *EFGH*?

(A) 25 in.^{2} (B) 416 in.^{2}

(C) 84 in.^{2} (D) 42 in.^{2}

(E) 68 in.^{2}

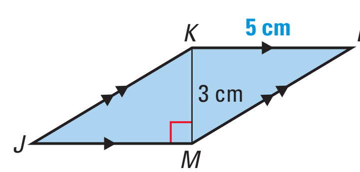


61. MULTIPLE CHOICE What is the area of parallelogram *JKLM*?

(A) 12 cm^2 (B) 15 cm^2

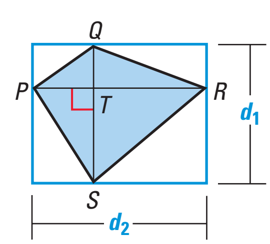
(C) 18 cm^2 (D) 30 cm^2

(E) 40 cm^2



★ Challenge

62. *Writing* Explain why the area of *any* quadrilateral with perpendicular diagonals is $A = \frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of the diagonals.



EXTRA CHALLENGE
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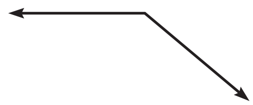
MIXED REVIEW

CLASSIFYING ANGLES **State whether the angle appears to be *acute, right,* or *obtuse*. Then estimate its measure.** **(Review 1.4 for 7.1)**

63. 

64. 

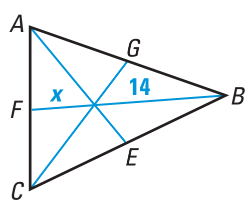
65. 

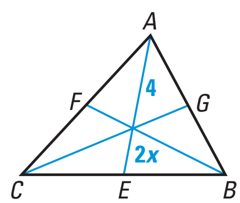
PLACING FIGURES IN A COORDINATE PLANE **Place the triangle in a coordinate plane and label the coordinates of the vertices.** **(Review 4.7 for 7.1)**

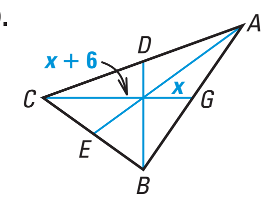
66. A triangle has a base length of 3 units and a height of 4 units.

67. An isosceles triangle has a base length of 10 units and a height of 5 units.

USING ALGEBRA **In Exercises 68–70, $\overline{AE}$, $\overline{BF}$, and $\overline{CG}$ are medians. Find the value of *x*.** **(Review 5.3)**

68. 

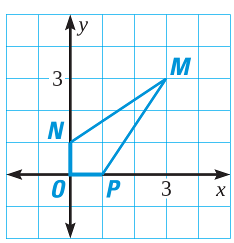
69. 

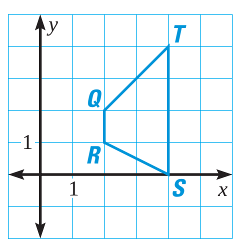
70. 

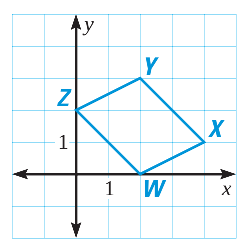
QUIZ 3

Self-Test for Lessons 6.6 and 6.7

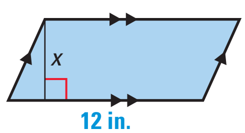
What special type of quadrilateral is shown? Give the most specific name, and justify your answer. **(Lesson 6.6)**

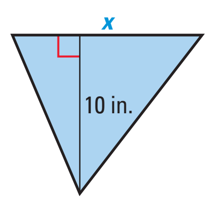
1. 

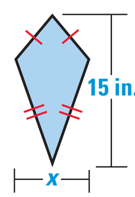
2. 

3. 

The shape has an area of 60 square inches. Find the value of *x*. **(Lesson 6.7)**

4. 

5. 

6. 

7. **GOLD BULLION** Gold bullion is molded into blocks with cross sections that are isosceles trapezoids. A cross section of a 25 kilogram block has a height of 5.4 centimeters and bases of 8.3 centimeters and 11 centimeters. What is the area of the cross section? **(Lesson 6.7)**